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What is Inquiry-based Mathematical Learning?

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On January 8, 2002, President George W. Bush signed the No Child Left Behind Act updating the Elementary and Secondary Education Act (Klein, 2015). The federal law made schools accountable for the education and learning they provided for their students. This accountability ensured that all students would be academically ready for college after graduating high school. They created the Common Core State Standards (CCSS) and the Standards for Mathematical Practices (SMPs) as a guide to help teachers prepare all of their students mathematically for college. They designed standardized testing to ensure that schools were teaching these standards in their classrooms. Although this law had great intentions and provided better education guidance on what is expected of teachers, it damaged the United States education system.

How did it damage the education system? If schools did not provide grade-level or higher learning outcomes for most of their students, then those schools would be defunded and shut down. How the schools reacted to the punishment of being defunded and shut down, created a ripple effect of damage to student learning. This kind of pressure on schools and students puts more focus on students getting good grades than focusing on effort. Lack of effort created bad habits and poor study skills, especially in mathematics.

In a world where technology creates lots of distractions and provides easy access to answers to math problems, students became great at getting good grades but developed horrible math habits. As a current substitute teacher, college student, and a previous college math tutor, I witnessed these distractions and horrible math habits in my interactions with students in our educational systems. For those who don’t believe me, a recent study states that math proficiency in the U.S. is 38%, and 50% in Washington state (Public School Review, 2023). Despite the changes and great efforts our teachers in public schools have made, teacher-centered classrooms at all levels of math are losing their effectiveness. Teacher-centered classrooms are losing their effectiveness at all levels of math because teachers do not know what inquiry-based mathematical learning is, how it works in a classroom, how it relates to the 5 practices, and how to balance their love for mathematics with the love for their students. When teachers change their teacher-centered classrooms to inquiry-based classrooms they may provide a more effective learning environment for all levels of mathematics.

Inquiry-based Mathematical Learning

To help teachers provide a more effective learning environment for all levels of mathematics, they should know what inquiry-based mathematical learning is. I gained a lot of information about inquiry-based mathematical learning here at EWU from my Senior Capstone class, MTED 490A, by researching articles, practicing the research in the classroom, and observing my mentor teacher. The information I gained about inquiry-based mathematical learning is that there are misconceptions and mistakes teachers make about it. I know this because I had some misconceptions and made mistakes about inquiry-based mathematical learning and found that it does not have the students blindly explore until they solve the problem, it is not a discussion guided by the students, it is not having the students show and tell their work, and it is not the teacher asking students lots of questions while they give direct instruction.

Inquiry-based mathematical learning is more student-centered learning and provides students with additional opportunities to practice the SMPs. It provides more chances with the SMPs because those opportunities are teacher-guided in the classroom. It is teacher-guided because the *teacher is responsible for creating an intellectual environment where serious mathematical thinking is the norm* (Beto, 2004). When serious mathematical thinking happens it creates *an engineer of learning environments in which students actively grapple with mathematical problems and construct their own understandings* (Smith, M. S., & Stein, M. K., 2018). If teachers learn that inquiry-based mathematical learning is where students actively explore mathematical problems and create their own understanding, then their classroom may provide a more effective learning environment for students at all levels of mathematics.

Inquiry-based Mathematical Learning Classroom

Providing an inquiry-based mathematical learning environment that is effective for their students may be challenging for teachers. Learning how an inquiry-based learning environment works in a classroom is the first step teachers can make to provide effective learning. Teachers must first create a new classroom setup where students can work in small groups, and the teacher can 1) *Teach by walking around*, 2) Have *a desktop code, strictly enforced*, and 3) have a *No shouted answers* rule (Johnson, 1994). When I used this classroom setup, it made classroom management easier, and whole class discussions more enriching.

Once the teacher sets up their classroom for inquiry-based mathematical learning, they can organize their lesson into three phases. The introducing phase, the exploring phase, and the summarizing phase. During the introduction phase, the teacher addresses *the whole class with the goal of helping the students understand what they are to investigate, explore, or solve* (Burns, 2022). This is also a time when the teacher shows an example of how to write information down, revisit prerequisites, and engage students. Next, is the exploring phase where students work together in small groups to create a solution to the math problem, activity, or task. During the exploring phase, a teacher should *observe the interaction, offer assistance when needed,* and *provide an extension for groups that finish* faster (Burns, 2022). Summarizing is the final phase where students discuss, present, and explain their strategies. Some tips on what to do during this phase are, to

> *have students speak one at a time; direct groups to choose a spokesperson to report to the entire class; encourage students to respond to each other's comments; record data on the board as it is presented* (Burns, 2022).

The teacher needs to make sure not to summarize for the students, making sure it is the students who are summarizing. When a teacher learns how to organize an inquiry-based mathematical lesson into these three phases, then they may provide an effective learning environment for students at all levels of mathematics.

5 Practices

Understanding the three phases of an inquiry-based lesson will help a teacher know how an inquiry-based lesson relates to the 5 Practices. It relates to the 5 Practices in which there are also three phases called the launch phase, the exploring phase, and the discussion and summarizing phase in which the three phases are basically the same (Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K., 2008). The 5 Practices are anticipating, monitoring, selecting, sequencing, and connecting. Before a teacher starts the launch phase in inquiry-based learning, they must first practice anticipating.

> *Through planning, teachers can anticipate likely student contributions, prepare responses they might make to them, and make decisions about how to structure students' presentations to further their mathematical agenda for the lesson* (Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K., 2008).

Anticipating students' responses first will help teachers be prepared for all of the other practices because the teacher needs to be prepared and plan for all three of the phases.

While students are busy during the exploring phase, the teacher will also be busy monitoring the students as they collaborate with their groups to solve the task. The teacher uses monitoring during the exploring phase as a time to assess students' understanding of the learning goal.

> *The goal of monitoring is to identify the mathematical learning potential of particular strategies or representations used by the students, thereby honing in on which student responses would be important to share with the class as a whole during the discussion phase* (Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K., 2008).

The teacher can also ask the students to explain their strategies to get better clarification on why they did the strategy and assess for any misconceptions. I found that using a monitoring chart was helpful because it helped me keep track of who was going to share their strategy and what learning goal the student was teaching. (See figure 1)

Strategy	Who and What	Order
Tasks 1 Basketball = most Golf ball = least		
Task 2 1 football ≈ 2 baseballs		
Task 3 Volleyball = 9 oz Football = 12 oz		
Task 4 Baseball = 5 oz Football = 9 oz Basketball = 16 oz		
Task 5 Football = 11 oz Volleyball = 7 oz Soccer ball = 13 oz		

Group 1:
Group 2:
Group 3:
Group 4:
Group 5:
Group 6:
Group 7:
Group 8:
Group 9:

(Figure 1)

The other nice thing about a monitor chart is that it can be used for selecting and sequencing parts of the 5 Practices. It also helps to keep track of who has shared during the week, so that different students get a chance to share their strategies. After monitoring the students' mathematical reasoning of their strategies, the teacher then selects students or groups who will share their strategies during the whole class discussion. Selecting students with a purpose to share, will help keep the discussion focused on the learning goal and prevent the students from just "show and telling" instead of having a meaningful discussion. The students might not figure out all of the strategies for the learning goal. If this happens, the teacher *can introduce a particularly important strategy that no one in the class has used by sharing the work of students from other classes or offering one of his or her own for the class to consider* (Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K., 2008).

I found that selecting students to share their strategies was challenging at first because a lot of the students were scared to share, but after they figured out it was a safe place to share their ideas they began to volunteer. This is where teachers need to plan the order students share by sequencing before and during the discussion and connection phase. Sequencing will help teachers guide the discussion and decide when is a good time to allow a student to share at random. By making *purposeful choices about the order in which students' work is shared, teachers can maximize the chances that their mathematical goals for the discussion will be achieved* (Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. 2008). After the mathematical goals for the discussion have been achieved, the teacher wraps up the lesson with the connecting practice part of the 5 Practices.

Connecting is how the teacher synthesizes the students' presentations of strategies with the learning goals. Connecting aims *to have student presentations build on each other to develop* *powerful mathematical ideas* (Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. 2008). Connecting those mathematical ideas shift the discussion from separate presentations to reflections of learning and thinking. It can help students connect differences and similarities between the two strategies. It can also influence the students to reflect on their strategies. Reflecting on mathematical ideas and how they connect with the 5 Practices may help teachers provide an effective learning environment.

I have experienced an inquiry-based lesson in my classes here at EWU and made lots of math connections, but I guess I did not realize how the professor was implementing this type of teaching strategy. Learning about the 5 Practices helped me understand what a teacher is doing during an inquiry-based lesson. After learning about the 5 Practices, I reflected on how I learned mathematics in a teacher-directed classroom growing up attending public schools. I made lots of mathematics connections and found that I practiced most of the SMPs in a teacher-directed learning environment and at home while I completed my math assignments. In fact, during my Senior Capstone class, I had the opportunity to observe my mentor teacher who is an excellent math teacher. After observing my mentor teacher's teaching strategies, I realized that my mentor teacher practices some of the 5 Practices.

Every great mathematics teacher will always solve the math problems first before teaching the steps on how to solve them to the class even if they already know it. This is what my mentor teacher did. Which is part of the anticipating practice of the 5 Practices. My mentor teacher also has taught mathematics for many years and is considered a master teacher because they have seen students make many mistakes or come up with many different strategies. Having this knowledge puts my mentor teacher in a place where they understand what the student is talking about when they explain their mathematical reasoning. I witnessed my mentor understand what a student was explaining during the discussion part of my lesson. I did not understand what the student meant, I knew it was incorrect, but my mentor teacher understood that the student was at the beginning stages of the math learning objective.

For the monitoring practice of the 5 Practices, my mentor teacher had a time when the students would work on the mathematic problems after receiving direct instruction on the prerequisites and example problems for the math assignment. My mentor teacher would walk around the room and help guide students to think about ways they could solve the problems. This is also a form of tutoring and also a way of assessing student learning because it gives the teacher a grasp of what part of the assignment the students understand and what part of the assignment the students are not making connections with. The problems my mentor teacher gave the students were high-cognitive demand questions that helped the students practice some of the SMPs, and they did wrap up their lesson by helping the students make mathematical connections with the learning goal. If teacher-directed learning can help students practice some of the SMPs, use high-cognitive demand questions, and help students make connections, then what is its problem with student proficiency in mathematics?

Some of the problems math teachers face in a teacher-directed classroom are the lack of student motivation. Once a student has identified themselves as someone who can’t do the math and has a fixed mindset about it, they lack the motivation and confidence to do the work in class and/or at home. This lack of motivation can cause a student to tune the math teacher out during the lesson. It can also cause the student to disbelieve they can do the work and can cause them to misbehave and disrupt the class. What makes matters worse is that the ripple effects of the No Child Left Behind Act still linger in our schools.

It still lingers in our schools because there is still the importance of getting good grades over putting forth effort. The importance of getting good grades leads to cheating for students who haven't been taught that effort is more important than getting good grades. As a substitute teacher, I have witnessed students copying the answers to an assignment without making connections. I have also witnessed this at a college level. At the middle-school level, students think they are helping their friends by giving them the answers, but there is no mathematical learning when this happens. The advancement in technology is what made my mentor teacher change from grading assignments to grading the students' efforts because students were just copying the math steps instead of making meaningful connections.

This is a problem. After all, when students get to a college level of math, they struggle with learning math because they have no connections to build from. I know this because when I tutored college students, I first taught the students the prerequisite math connections before I could help them make new connections. Some students not linking the prerequisite math connections before they enter college reveals that teacher-directed classroom environments at all levels of math are not as effective as they should be. Most students can make mathematical connections because they can develop plasticity. A fixed mindset may prevent students from believing their brains cannot make mathematical connections.

> *When math greats are presented to students as born geniuses, it puts students into a fixed mindset; whereas when math geniuses are presented as people who loved and devoted themselves to math, it conveys a growth mindset to students*. (Boaler, 2015; Dweck, 2008)

To help students with a growth mindset in mathematics, teachers must inspire students that they have the potential to develop plasticity when learning mathematics. They can inspire students by finding a balance between their love for mathematics and their love for students.

Balancing Love for Mathematics and Love for Students

By learning how to balance their love for mathematics with their love for their students, teachers may be able to provide an effective learning environment for their students. Balancing love in a mathematics classroom will help students *to express powerful mathematical understandings in personally meaningful ways* (Bleiler-Baxter 2022). As a substitute teacher, I have to teach whatever the teacher has planned for me to teach, and because I love mathematics and know the importance of what the teacher is asking me to teach, I tend to focus on mathematics more than the students' voice or volunteering to share. As a student, time being my biggest challenge, I found myself either trying to teach too much math or trying to share too much student work in a small amount of time. The students lacked understanding if I tried to have too many students share, our discussion lacked deep conversation and mathematical learning and was more like show-in-tell instead. Learning from my mistakes as a substitute and a student made me think about what changes I needed to make and how I could use the five practices in my classroom. The more I balanced the five practices when I taught, the more I balanced my love for mathematics and students.

I was fortunate to have a well-experienced mentor teacher who not only has taught math for many years but also received his master's degree in secondary mathematics education at EWU. I learned that he has a love for mathematics and has earned the compliment of being called a "Math Person" which basically means

valuing the beauty of mathematics and honoring the norms of the discipline. It values the

> *ways that mathematical ideas are precisely and concisely shared as well as the processes that mathematicians use to come to understand mathematical concepts. It recognizes that norms and processes sometimes change over time and that the term mathematicians includes a vast array of different types of people who do mathematics professionally within the discipline* (Bleiler-Baxter 2022).

My mentor teacher is great at accessing what the students are lacking and teaching them the mathematical connections they need to know. He starts his lessons with beginning mathematical concepts and then builds upon those ideas as the students experience more complex mathematical learning. He also does not have one set way of teaching students; he changes the lessons up. This gives his students a variety of ways of learning mathematics.

Another thing I have learned from my mentor teacher is that he loves and cares about his students' mathematical learning. The time he takes to listen to each student's mathematical reasoning on what their test mistakes were and how they fixed them tells me that he loves and cares about his students. Although my mentor teacher loves his students, his love for them does not fully,

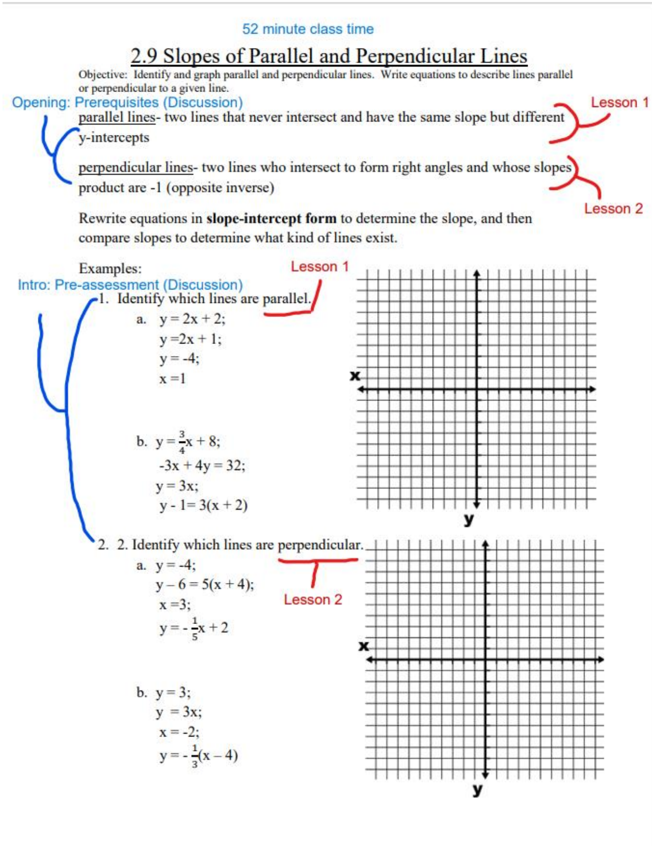
> *acknowledges that students should be free to come to understand mathematics in ways that are respectful of their agency as human beings. It requires that teachers provide students with the time, space, tools, and challenges to grow in their understanding of mathematics* (Bleiler-Baxter 2022).

His love does not fully acknowledge students' ways to solve math problems because he uses teacher-directed instruction. If he used more student-centered instruction, he would be providing time, space, tools, and challenges to help students grow in their own understanding of mathematics. I believe, being the math person he is, that my mentor teacher would use student-centered instruction in his classroom. He would use student-centered instruction, but changing his whole curriculum would be very challenging.

It would be very difficult because there are learning objectives that have to be taught by certain dates and making a big change may prevent him from meeting those expectations. Not meeting those expectations may look bad, but what if the changes he could make did not have to be so big? They do not have to be so big because he can still use the same curriculum. During my research in his class, he would give me notes and homework worksheets. I would use those worksheets to create a student-centered lesson from the perspective of the articles I read and the 5 practices. Sometimes I created my questions and other times I just used the notes worksheet and organized them to fit the three phases of the 5 practices.

For example, the first worksheet at the top is where the teacher refreshes the prerequisites by reading the math vocabulary with definitions out loud. (See figure 2) For my student-centered lesson, I would first start the opening by using an anchor text displayed on a PowerPoint for the students to observe and begin a whole class discussion about parallel lines asking the students what they observe about the picture and what mathematical ideas they see. (See figure 3) If the students point out that the railroad tracks are parallel, I would ask them how they know they are parallel having students pull previous knowledge from their memory to answer the questions. Then showing them the next slide to reinforce what students said or if no one points out the parallel connection, I would guide them to it and show them the next slide. (See figure 4)

The second part of the first worksheet would start my intro part of the lesson. This is also a discussion where I show a new slide from the PowerPoint and ask the students which lines are parallel. (See figure 5) The students would observe the lines written in the slope-intercept form on the slide and engage in a whole class discussion about why they think which lines are parallel.


52 minute class time

2.9 Slopes of Parallel and Perpendicular Lines

Objective: Identify and graph parallel and perpendicular lines. Write equations to describe lines parallel or perpendicular to a given line.

Opening: Prerequisites (Discussion)

Lesson 1

parallel lines- two lines that never intersect and have the same slope but different y-intercepts

perpendicular lines- two lines who intersect to form right angles and whose slopes product are -1 (opposite inverse)

Lesson 2

Rewrite equations in **slope-intercept form** to determine the slope, and then compare slopes to determine what kind of lines exist.

Examples:

Lesson 1

Intro: Pre-assessment (Discussion)

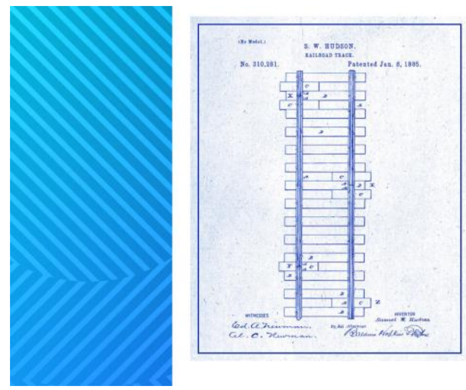
1. Identify which lines are parallel.
   a. $y = 2x + 2$; $y = 2x + 1$; $y = -4$; $x = 1$
   b. $y = \frac{3}{4}x + 8$; $-3x + 4y = 32$; $y = 3x$; $y - 1 = 3(x + 2)$

2. 2. Identify which lines are perpendicular.

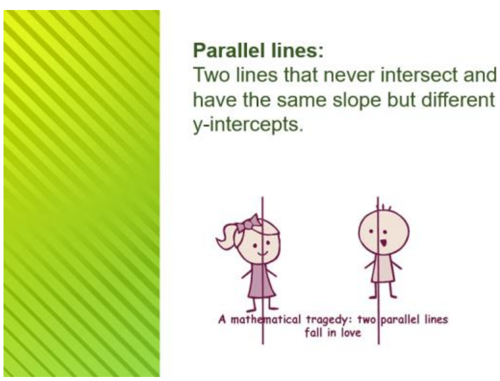
Lesson 2

   a. $y = -4$; $y - 6 = 5(x + 4)$; $x = 3$; $y = -\frac{1}{5}x + 2$
   b. $y = 3$; $y = 3x$; $x = -2$; $y = -\frac{1}{3}(x - 4)$

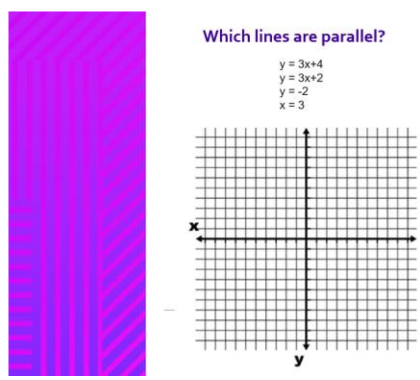
(figure 2)



(figure 3)



(figure 4)

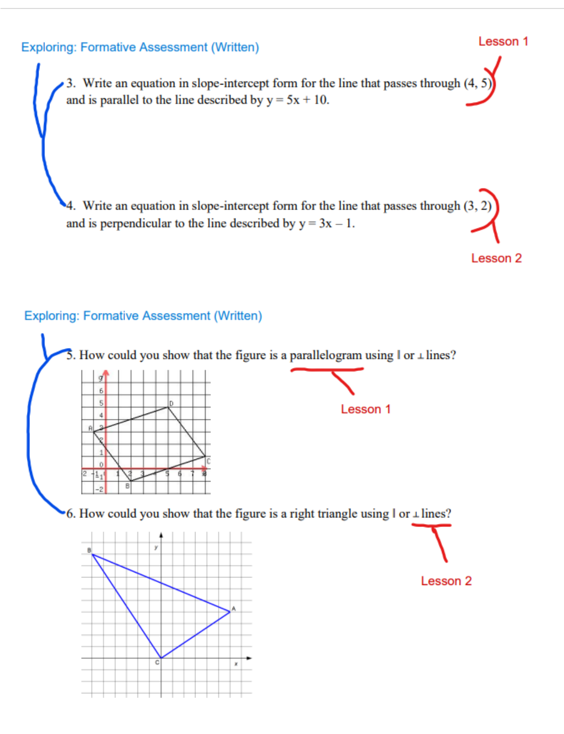


(figure 5)

This is also a pre-assessment because I have to make sure the students have enough tools to complete the exploring part of the lesson. After our discussion, I would briefly teach any prerequisite tools they did not mention. Then move on to the second page of the notes worksheet and begin the exploring part of the lesson. (See figure 6)

The exploring part of the lesson is where the students are working in small groups collaborating on how to solve the problems on the worksheet. During the exploring phase, I am monitoring and assessing to see who will be sharing their strategies that teach the learning goals that I planned for the students to learn. I would also be using a monitor chart to organize which student or group would be sharing their strategy for what learning goal and when. (See figure 7) If the students get stuck and cannot seem to figure out how to solve the problems, then I would either have a student who has solved the problem share and explain their strategy to the class or if everyone is stuck I would share the slides I prepared to help them advance. (See figure 8) After the exploring phase is over and I have chosen enough students to present their work, we start the whole class discussion.

(figure 6)

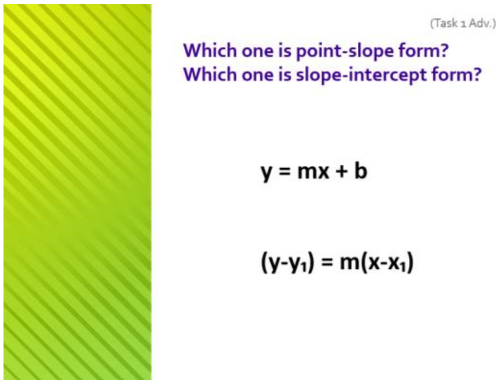


(figure 7)

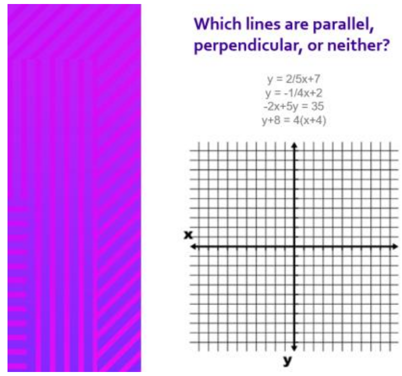
Strategy	Who and What	Order
Task 1 (y-7) = 3(x-6) y-7 = 3x – 18 +7 = +7 y = 3x – 11 m = 3		
Task 2 Line BC → y = 4/3x + 16/3 parallel to Line AD → y = 4/3x - 3		
Task 2 Line BA → y = -3/4x – 3 Parallel to Line CD → y = -3/4x + 38/4		
Something Different		

Group 1:
Group 2:
Group 3:
Group 4:
Group 5:
Group 6:
Group 7:
Group 8:
Group 9:
Group 10:
Group 11:

(figure 8)



(figure 9)

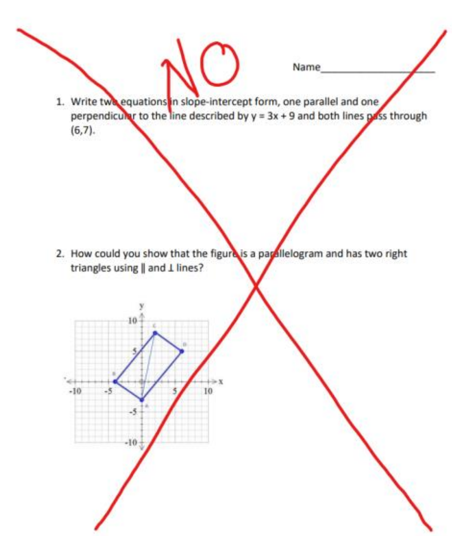


As students present their strategies, I help facilitate a whole class discussion and make sure they are discussing the learning objects. It's important to ask the students why they did that certain strategy, and even if it is a correct method, ask if anyone disagrees or agrees and why. Asking if anyone disagrees or agrees with the correct method, sets the norm for all discussions because it helps students share any misconceptions they may think are correct or incorrect. The student or group may be sharing the incorrect method, and asking if anyone disagrees will help students discuss why they disagree and become a teachable moment. When this part of the discussion is a norm, the student sharing an incorrect method becomes familiar with discussing students' agreements and disagreements. After everyone has shared and time permits it, ask if anyone used a different strategy or method to solve problems. This helps students develop flexibility with how they solve problems and teaches them that there is not just one way to solve problems.

The last part of a student-centered lesson is the connecting phase. I created a slide for the connecting part of my lesson because it is a whole class discussion. Asking the students which lines are parallel, perpendicular, or neither revisits and connects the learning objective and it moves the students to the beginning of the next lesson about perpendicular lines. (See figure 9) This completes the lesson, unless the teacher chose to put a sticky note on the students desk while monitor and assessing the students work during the exploring phase. Then the teacher would have the students put one thing they liked about today and one thing they did not understand on the sticky note and stick it on the whiteboard as they leave the classroom.

One thing I strongly suggest not to do, when transforming the teacher-directed notes worksheets into a student-centered lesson, is trying to put all of the information from the worksheet into the lesson. I tried to create a student-centered lesson plan with both parallel and perpendicular learning objective and that did not go very well. It did not go well because I had two openings and two introductions that took longer than what I planned. I also tried to combine questions into one question by putting a parallel and perpendicular questions in the same question. This was very confusing to the students and was a major failure when I taught this lesson. (See figure 10) When I revised the lesson by breaking it up into two lessons, one for parallel and one for perpendicular. (See figures 2 and 6) I also created a separate worksheet for each lesson. (See figure 11) Breaking up the notes worksheets into two separate lessons would give the students enough time to explore and have a meaningful discussion.

(figure 10)



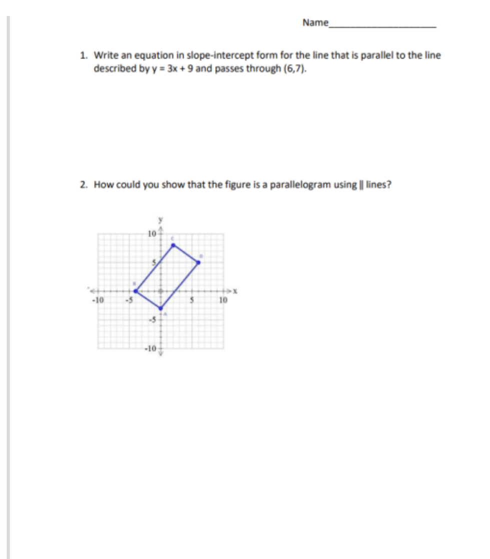
NO

Name________________

1. Write two equations in slope-intercept form, one parallel and one perpendicular to the line described by y = 3x + 9 and both lines pass through (6,7).

2. How could you show that the figure is a parallelogram and has two right triangles using ∥ and ⊥ lines?

(figure11)



Name________________

1. Write an equation in slope-intercept form for the line that is parallel to the line described by y = 3x + 9 and passes through (6,7).

2. How could you show that the figure is a parallelogram using ∥ lines?

(figure 12)

Name__________________________

Solve each of the following problems using numbers, a table, and a graph.

1. At Peterson Farms, Inc. on average, one medium apple fills the machine with 4oz of applesauce. How many ounces of applesauce will 48 apples make?

2. The machine takes 1 second to fill 8 (4oz) cups of apple sauce. How many seconds will it take the machine to empty 384 oz of applesauce?

There was one time that I was able to successfully create a student-centered lesson that had two learning objectives, discrete and continuous functions, from the worksheet my mentor teacher gave me. It was successful because I created a PowerPoint video, and I had the students watch and take notes on the night before my lesson. I only had two questions that had the students represent their learning in multiple ways. (See figure 12) I used one video for the opening but showed it as two parts to represent each learning objective and each question. If teachers feel that the lesson is not enough to practice for students, they could assign the homework worksheet to complete at home. Following this method I developed may help teachers with teacher-directed classrooms make an easier transition to a more student-centered classroom by providing inquiry-based mathematical learning for their students in which they would be balancing their love for mathematics with their love for their students.

Balancing their love for mathematics with their love for their students will create a classroom that promotes mindful mathematical learning. When teachers take the time to learn about what inquiry-based learning is, how it works in the classroom, how it relates to the 5 Practices, and how to balance their love for mathematics with their love for their students they will provide an effective learning environment at all levels of mathematics. I would really like to do more research on using the 5 Practices and what I learned in my Senior Capstone class. The reason why I would like to do more research is that I found classroom management to be way easier to manage, students were more engaged, and the lesson took the whole class time. By the time I taught my final lesson, which was observed by my supervisor, more students were willing to get up and share their strategies and I was able to get the timing down where all of the phases were completed. Doing more research on this type of student-centered math lesson will help me receive a greater grasp on how well the students are learning this way and I hope to practice this while I complete my final student teaching in the classroom.

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