TARDYS Quantifiers: Extracting Temporal and Reversible DYnamical Symmetries

Nhat Vu Minh Nguyen
Eastern Washington University

Arjendu K. Pattanayak
Carleton College

Andres Aragoneses
Eastern Washington University

TARDYS Quantifiers: Extracting Temporal and Reversible DYnamical Symmetries

Nhat Nguyen*, Arjendu K. Pattanayak**, and **Andrés Aragoneses***

* Department of Physics, Eastern Washington University, Cheney, WA 99004, ** Department of Physics and Astronomy, Carleton College, Northfield, MN 55057

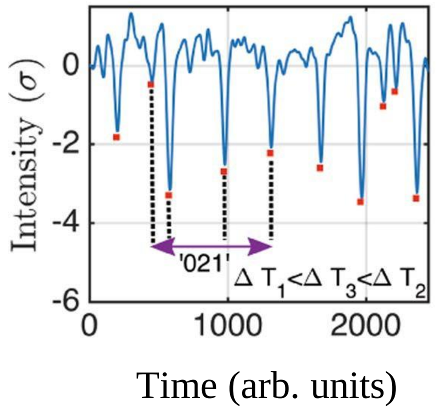
INTRODUCTION.

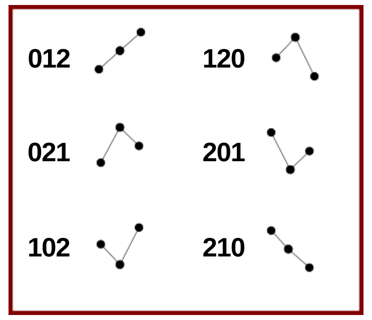
Complex dynamical systems display a broad variety of behaviours depending on parameters and initial conditions. When the dynamical equations are unknown, the challenge is to characterize them by identifying temporal structures in the dynamical observations, specifically in systems with observational or experimental noise.

One approach is to use approximate symmetries in the dynamics based on ordinal patterns. These compare consecutive events to transform the time series into a sequence of patterns, also known as words.

TIME SERIES ANALYSIS OF A PHOTONIC NEURON.

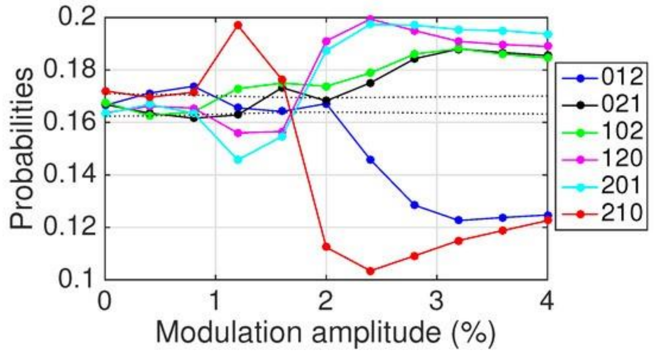
Photonic neuron: A semiconductor laser with external optical feedback. It replicates the features of the spiking dynamics of neurons. We explore the dynamics of a photonics neuron under periodic modulation.





Words of dimension D=3.

The words probabilities of a photonic neuron as a function of the modulation amplitude present some symmetries. These are approximate symmetries as they are computed with ordinal patterns (relative time intervals):



$P_2 \approx P_3 \qquad P_4 \approx P_5$

$$|P_1 - \tfrac{1}{6}| - |(P_2 - \tfrac{1}{6}) + (P_3 - \tfrac{1}{6})| \approx 0$$

$$|P_6 - \tfrac{1}{6}| - |(P_4 - \tfrac{1}{6}) + (P_5 - \tfrac{1}{6})| \approx 0$$

TARDYS QUANTIFIERS: T_α, T_β, T_δ, T_ρ.

We define new complexity measures to extract the temporal and reversible dynamical symmetries of a complex system based on the natural approximate symmetries of the photonics neuron[1].

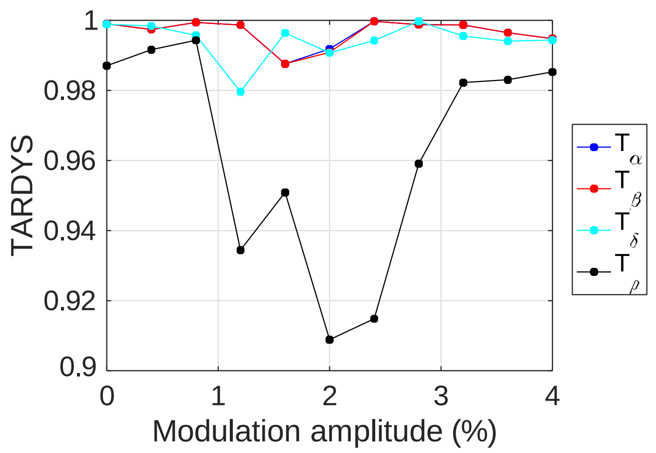
Symmetry of triad P_1, P_2, P_3: $T_\alpha = 1 - |w_1 - w_\alpha|$

Symmetry of triad P_6, P_4, P_5: $T_\beta = 1 - |w_6 - w_\beta|$

Sym. of clusters $P_2, P_3; P_4, P_5$: $T_\delta = 1 - |P_2 - P_3| - |P_4 - P_5|$

Reversibility: $T_\rho = 1 - |P_1 - P_6| - |P_2 - P_4| - |P_3 - P_5|$

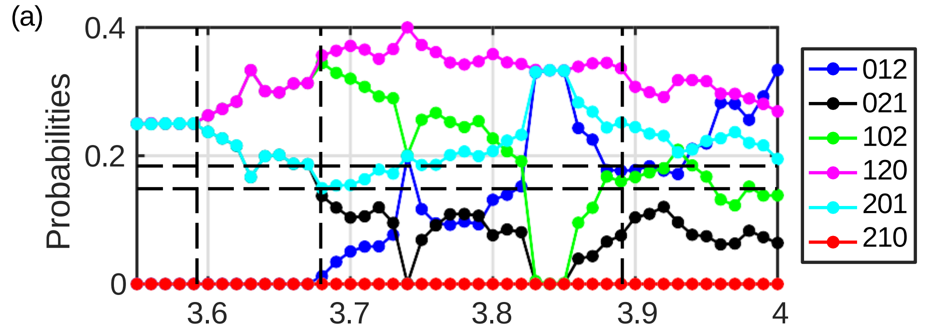
with: $w_1 = |P_1 - \frac{1}{6}| \quad w_\alpha = |(P_2 - \frac{1}{6}) + (P_3 - \frac{1}{6})| \quad w_6 = |P_6 - \frac{1}{6}| \quad w_\beta = |(P_4 - \frac{1}{6}) + (P_5 - \frac{1}{6})|$



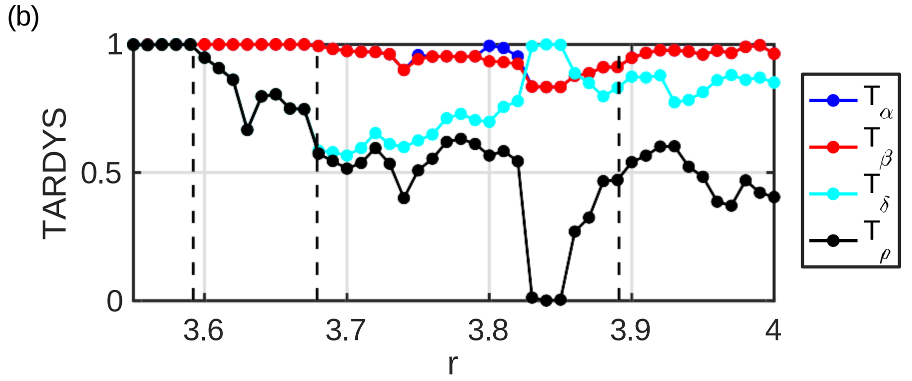
THE LOGISTIC MAP.

We apply the new TARDYS quantifiers to the theoretical iterative logistic map to explore its internal symmetries.

$$x_{i+1} = r x_i (1 - x_i)$$



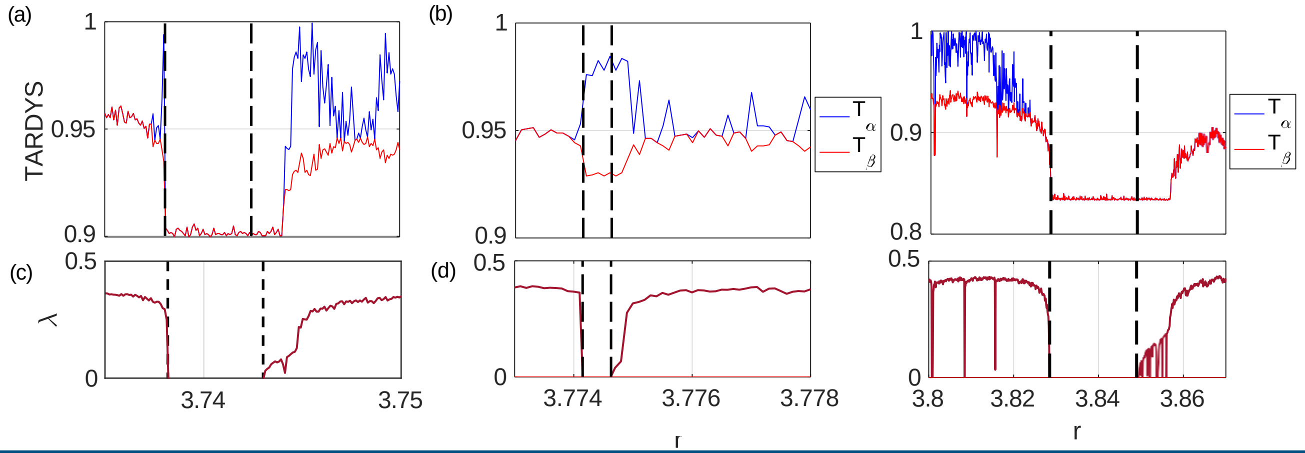
The logistic map presents 4 clear chaotic regions with different dynamics[2].



TARDYS quantifiers capture some transitions in dynamics.

PREDICTIVE POWER

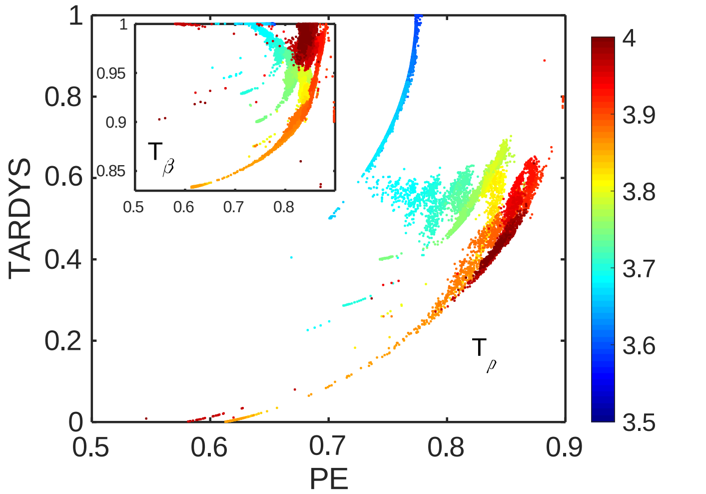
TARDYS T_α and T_β versus r at the edge of the windows of periodicity (top). Lyapunov exponent versus r (bottom).



TARDYS VERSUS PERMUTATION ENTROPY

Combining TARDYS with other parameters, here Permutation Entropy (PE) reveals more information. It helps differentiate and classify families of chaos.

$$PE = -\frac{1}{log(D!)} \sum_i^{D!} P_i \, log(P_i)$$



CONCLUSIONS

TARDYS quantifiers extract hidden dynamical symmetries and identify transitions in dynamics.

Natural (approximate) symmetries of a photonics neuron are informative in other systems.

TARDYS can forecast sharp chaos-regular transitions.

TARDYS with PE allow to classify families of chaos.

BIBLIOGRAPHY

(1) Nguyen et al., *Photonics*, **9**, 938 (2022).

(2) Spichak and Aragoneses, *Chaos, Solitons, and Fractals,* **154** (2022)